Fluid Statics

In fluids at rest there are no shear stresses;

- only normal forces due to pressure are present.
- Normal forces produced by static fluids are often very important.

-For example,

they tend to overturn concrete dams,

burst pressure vessels,

and break lock gates on canals.

Obviously, to design such facilities, we need to be able to compute the magnitudes and locations of normal pressure forces. Understanding them, we can also develop instruments to measure pres sures, and systems that transfer pressures, such as for automobile brakes and hoists.

Fluid Statics

- The ***average pressure intensity*** *p is* the force exerted on a unit area. If *F* represents the total normal pressure force on some finite area *A,* while *dF* represents the force on an infinitesimal area *dA,* the pressure is

  $p = dF/dA$
- if the pressure is uniform over the total area, then
- $p = F/A$.
- *Units*

  psi(lb/in^2)

  psf(lb/ft^2

  Pa(N/m^2) or

  kPa (kN/m^2).

Fluid Statics

PRESSURE AT A POINT THE SAME IN ALL DIRECTIONS

Reasons:

- no tangential stresses can exist in a fluid at rest
- only forces between adjacent surfaces are pressure forces normal to the surfaces.

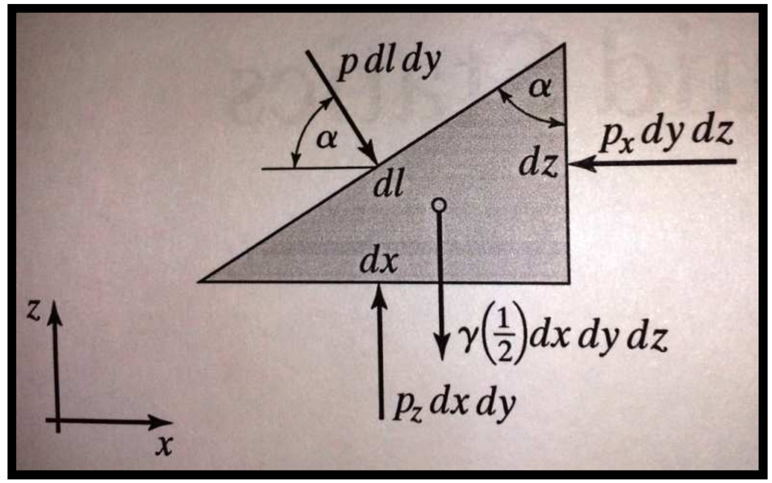
Proof:

a very small wedge-shaped element of fluid at rest whose thickness perpendicular to the

Fluid Statics

Proof:

a very small wedge-shaped element of fluid at rest whose thickness perpendicular to the plane of the paper is dy. Let p be the average pressure in any direction and let px and pz be the average pressures in the horizontal and vertical directions. forces in the y direction cancel each other. For equilibrium, the sum of the force components on the element in any direction must be equal to zero. in the x di-rection,



$\boldsymbol{p dl dy cos\alpha - p_x dy dz = 0}$.

Since $\boldsymbol{dz = dl cos a}$,

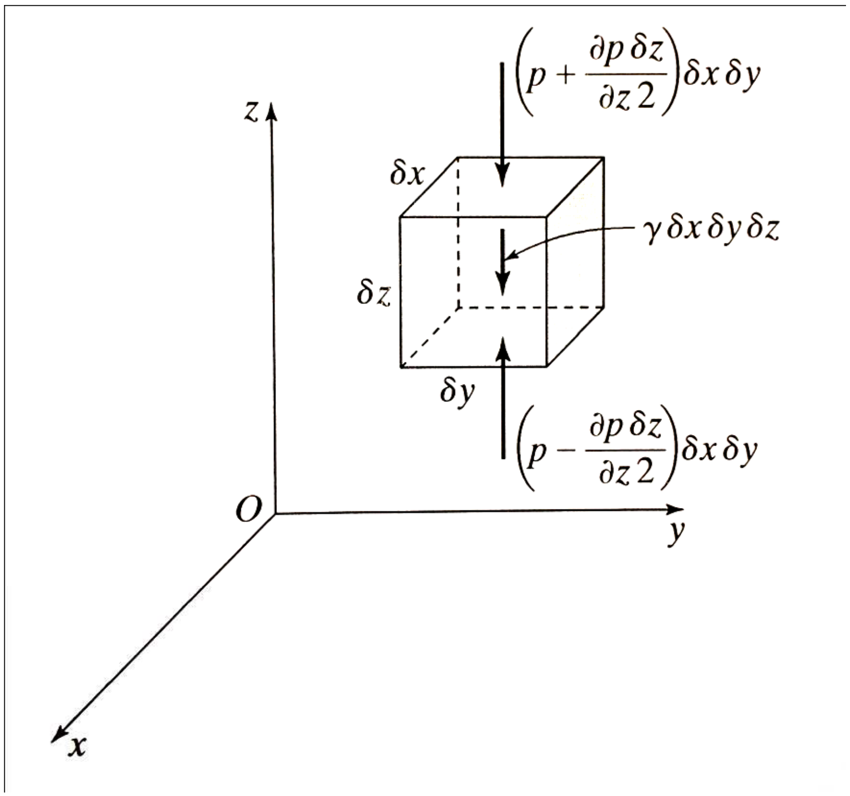
it follows that $\boldsymbol{p = p_x}$.

Similarly, in the z direction gives

$\boldsymbol{p_Z\, dx\, dy - p\, dl\, dy \sin \alpha - 1/2\, \Upsilon dx\, dy\, dz = 0}$.

The third term is of higher order than the other two terms and so may be neglected. It follows from this that $\boldsymbol{p = p_Z}$. We can also prove that $\boldsymbol{p = p_y}$ by considering a three-dimensional case. The results are independent of α; thus the pressure at any point in a fluid at rest is the same in all directions.

Fluid Statics



VARIATION OF PRESSURE IN A STATIC FLUID

Consider the differential element (or control volume) of static fluid shown in Fig. pressure at the center p.

The forces acting in the vertical direction are (a)

The body force, the action of gravity on the mass within the element, and (b) the surface forces, transmitted from the surrounding fluid and acting at right angles against the top, bottom, and sides of the element. summation of forces acting on the element in any direction must be zero. If forces are summed in the horizontal direction, that is, x or y, the only forces acting are the pressure forces on the vertical faces of the element. To satisfy

$$\sum F_y = 0 \quad \text{and} \quad \sum F_x = 0$$

the pressures on the opposite vertical faces must be equal. Thus for the case of the fluid at rest.

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y}$$

Fluid Statics

Summing forces in the vertical direction and setting the sum equal to zero,

$$\sum F_x = \left(p - \frac{\partial p}{\partial x}\frac{\partial z}{2}\right)\partial x\partial y - \left(p + \frac{\partial p}{\partial x}\frac{\partial z}{2}\right)\partial x\partial y - \gamma\partial x\partial y\partial z = 0$$

This results in $\frac{\partial p}{\partial z} = -\gamma$

since p is independent of x and y, this can be written as $\frac{dp}{dz} = -\gamma$ (3.2)

This is the general expression that relates variation of pressure in a static fluid to vertical position. The minus sign indicates that as z gets larger (increasing ele-vation), the pressure gets smaller.

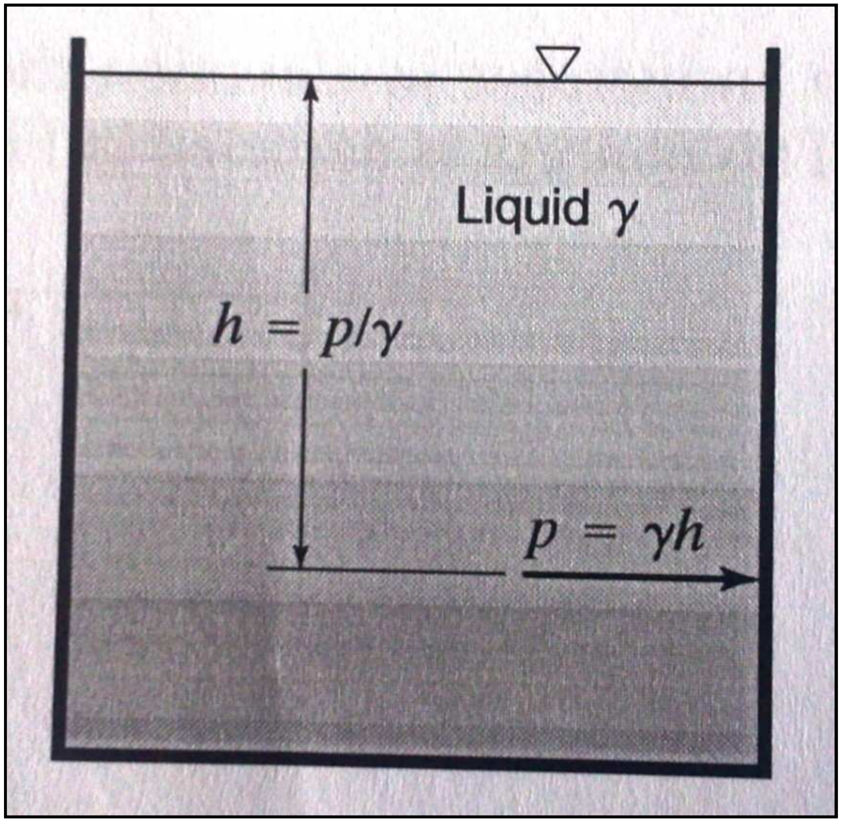
To evaluate the pressure anywhere in a fluid at rest, we must integrate Eqn 3.2 between appropriately chosen limits. For incompressible fluids (ϒ = constant), we can integrate Eq. (3.2) directly.

$$\frac{dp}{dz} = -\gamma; \quad dp = -\gamma\, dz; \quad \int_{p_1}^{p} dp = -\gamma \int_{z_1}^{z} dz$$

$$p - p_1 = -\gamma(z - z_1) \quad (3.3)$$

For compressible fluids, however, we must express y algebraically as a function of z or p if we wish to determine pressure accurately as a function of elevation

Fluid Statics



PRESSURE EXPRESSED IN HEIGHT OF FLUID

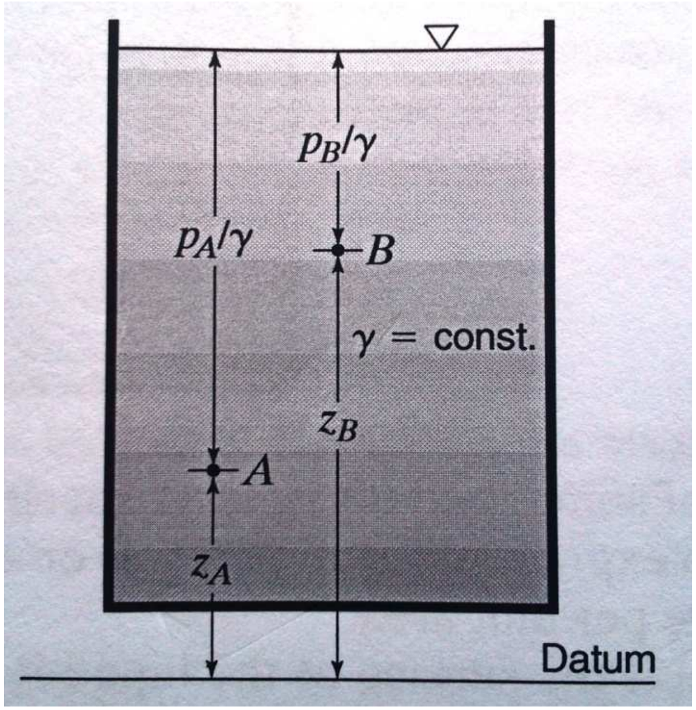
- Imagine an open tank of liquid with no pressure acting on its surface (Fig. 3.3), though in reality the minimum pressure upon any liquid surface is the pressure of its own vapor. Disregarding this for the moment, by Eq. (3.4), the pressure at any depth *h is p = yh.* If we assume *y* to be constant, there is a definite relation between p and h. That is, pressure (i.e., force per unit area) is equivalent to a height *h* of some fluid of constant specific weight y. Often we find it more con-venient to express pressure in terms of a height of a column of fluid rather than in pressure per unit area.
- Even if the surface of the liquid is under some pressure, we only need to convert this pressure into an equivalent height of the fluid in question and add this to the value of *h* shown in Fig. 3.3, to obtain the total pressure.

Thus we may relate pressure *p* to the height of a column of *any* fluid by the expression

$$h = \frac{p}{\gamma}$$

This relationship is true for any consistent system of units. When we express pressure in this way, in terms of a height of fluid, we commonly refer to it as ***pres-sure head***

Fluid Statics



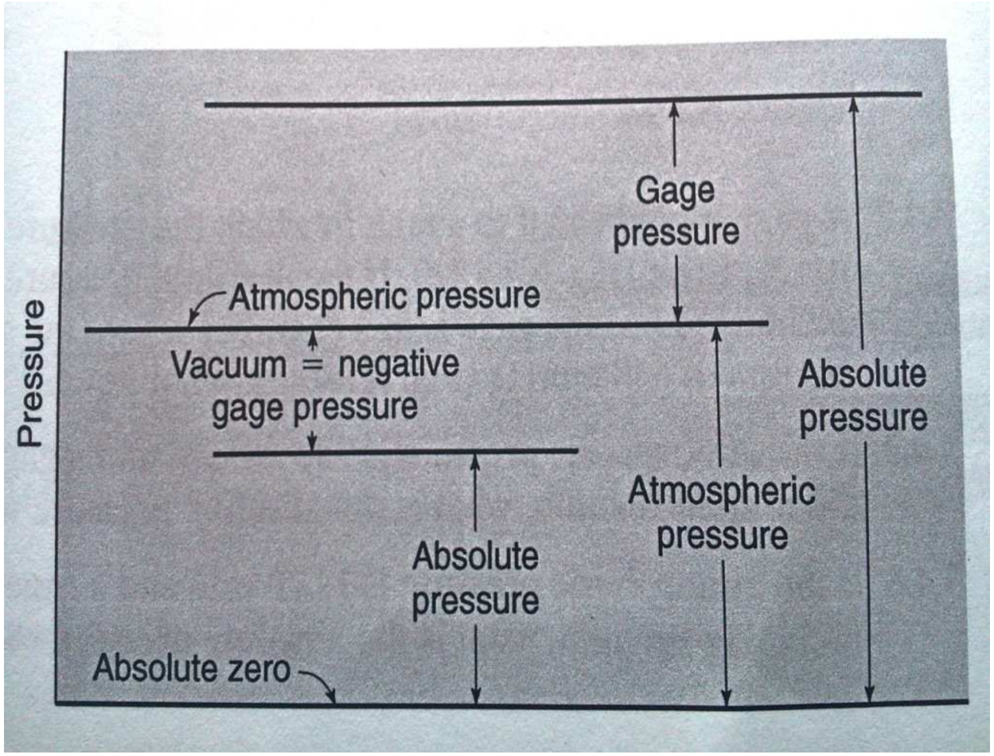
An important property follows from Eq. (3.3), which we can express as:

For Incompressible fluid:

$p/\Upsilon + z = p_1/\Upsilon + z_1$ = constant

This shows that for an incompressible fluid at rest, at any point in the fluid the sum of the elevation z and the pressure head p/Υ is equal to the sum of these two quantities at any other point. The significance of this statement is that, in a fluid at rest, with an increase in elevation there is a decrease in pressure head, and vice versa. This concept is depicted in Fig.

Fluid Statics



pressure relative to absolute zero is called ***absolute*** pressure; Pressure relative to atmospheric pressure is ***gage*** pressure.

If the pressure is below that of the atmosphere, we call it *a* ***vacuum.***

All values of absolute pressure are positive,

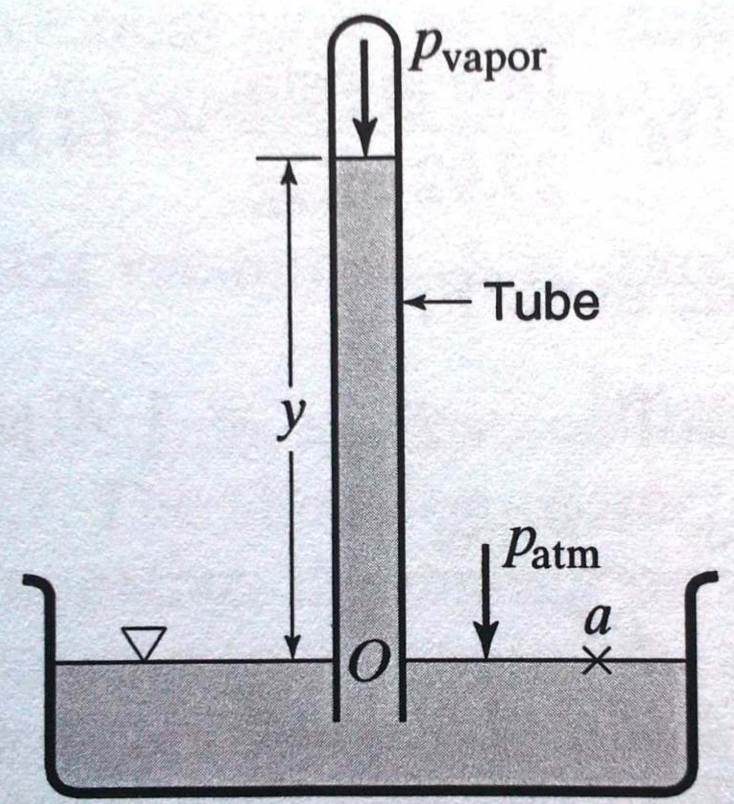
We can see from the preceding discussion that the following relation holds,

$p_{abs} = P_{atm} + P_{gage}$

where p_{gage} may be positive or negative (vacuum).

Fluid Statics

Measurement of Pressure



(*a*) Mercury barometer

Barometer

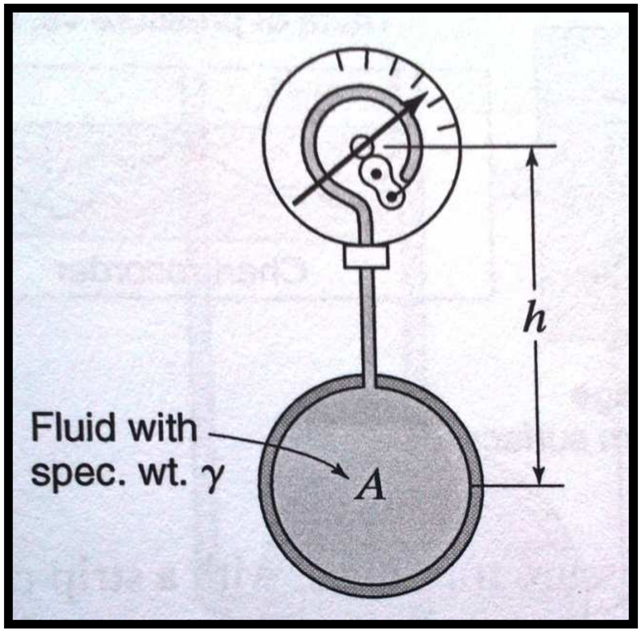
- We measure the absolute pressure of the atmosphere with a barometer.
- From the concepts developed in Sec. 3.2, we see that the pressure at 0, $p_o = p_a = p_{atm}$. But, from Eq. (3.4) and Sec. 3.2,

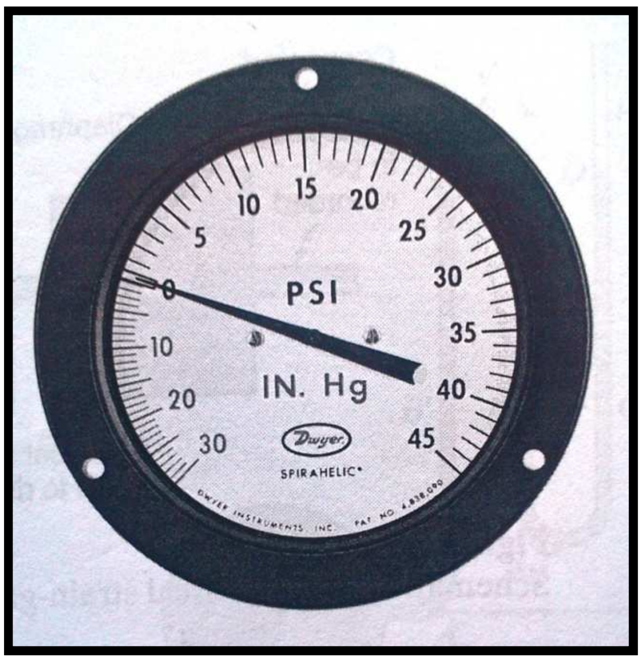
  $p_o = \Upsilon y + p_{vapor}$
- Because of the static equilibrium, we may equate the pressures at 0 to obtain

  $$p_{atm} = \Upsilon y + P_{vapor} \qquad (3.8)$$
- If the vapor pressure on the surface of the liquid in the tube were negligible, then we would have

  $P_{atm} = \Upsilon y$

  Mercury is used to keep the tube short.



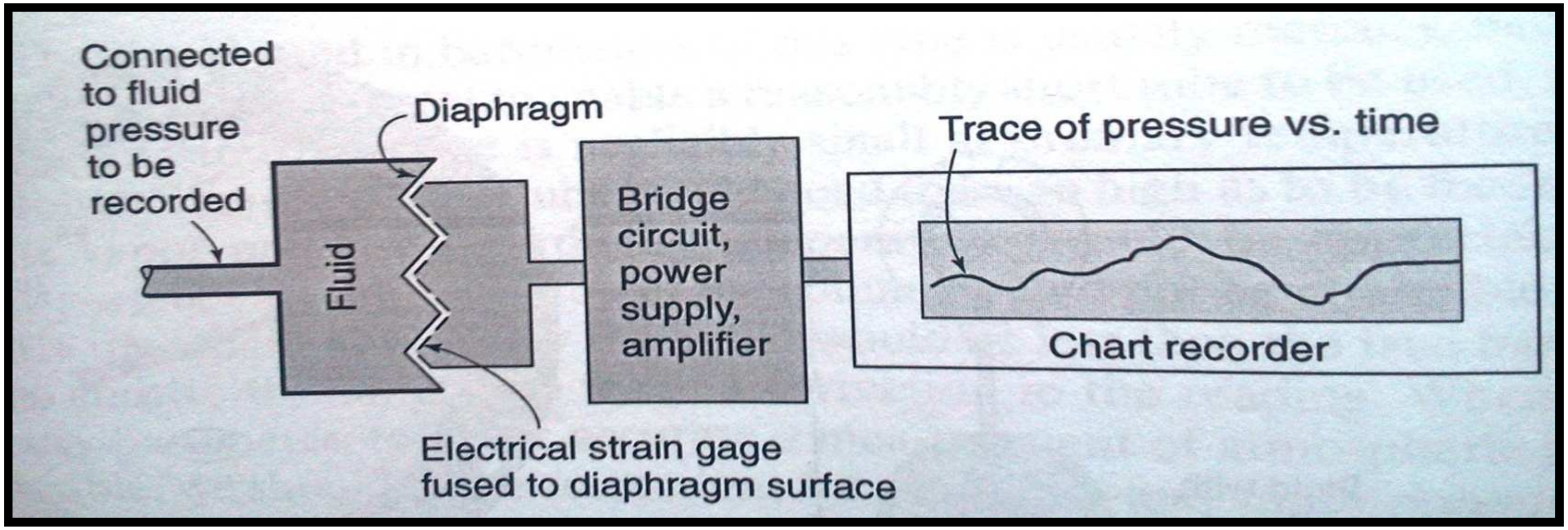


Bourdon Gage

- We commonly measure pressures or vacuums with the ***Bourdon gage***.
- When a pressure and vacuum gage is combined into one we call this *a* ***compound gage***.
- The pressure indicated by such gages is that at their centers. *If* the connecting piping is filled completely with fluid of the same density and if the pressure gage is graduated to read in pounds per square inch, as is customary, then

$$p_A(\text{psi}) = \text{gage reading (psi)} + \Upsilon h/144$$

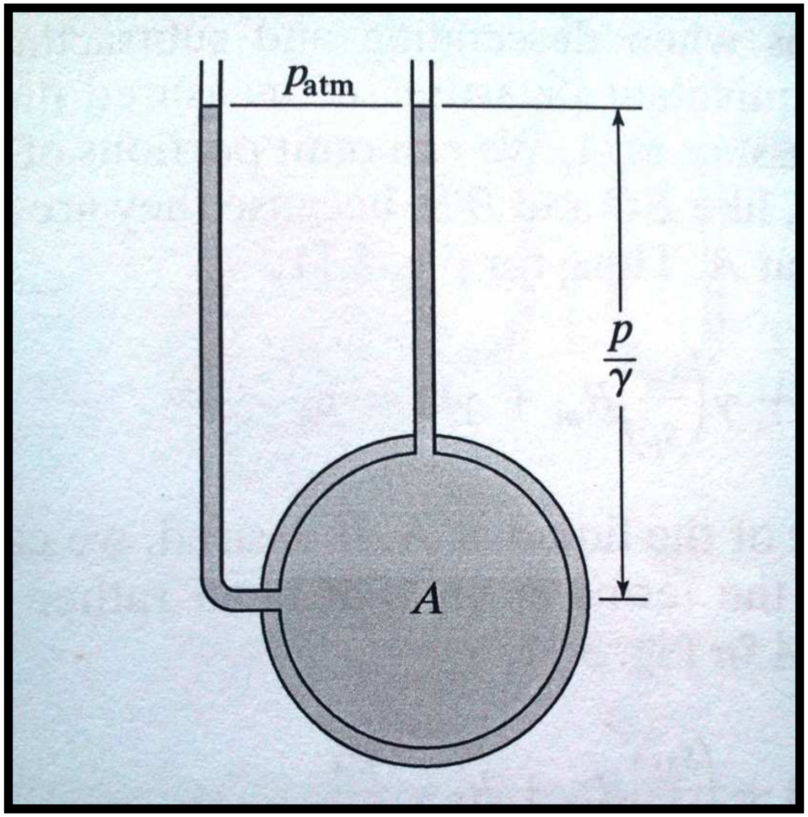
Fluid Statics



Pressure Transducer

- *A* ***transducer*** is a device that transfers energy (in any form) from one system to another.
- A Bourdon gage is a mechanical transducer
- An ***electrical pressure transducer*** converts the displacement of a mechanical system (usually a metal diaphragm) to an electric signal, either actively if it generates its own electrical output or passively if it requires an electrical input that it modifies as a function of the me-chanical displacement.
- As the pressure changes, the deflection of the diaphragm changes. This, in turn, changes the electrical output, which, through proper calibration, can provide pressure.
- If we connect such a device to a strip-chart recorder we can use it to give a continuous record of pressure

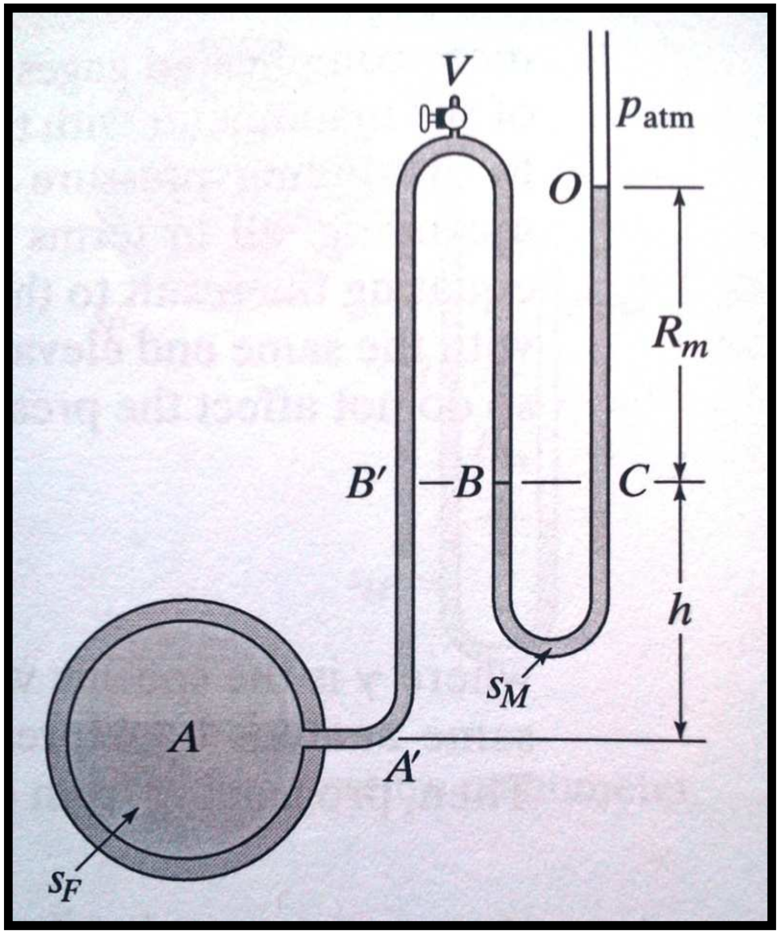
Fluid Statics
Measurement of Pressure



Piezometer Column

- A piezometer column is a simple device for measuring moderate pressures of liquids.
- It consists of a sufficiently long tube in which the liquid can freely rise without overflowing.
- The height of the liquid in the tube will give the value of the pressure head, p/Υ directly.
- To reduce capillary error the tube diameter should be at least 0.5 in (12 mm).
- open piezometer tube is too tall and cumbersome for use with liquids under high pressure and gases

Fluid Statics
Measurement of Pressure



Simple Manometer

- simple manometer or mercury U tube is a convenient device for measuring many pressures.
- Let s_M *be* the specific gravity of the *manometer* fluid and s_F as the specific gravity of the *fluid* whose pressure is being measured. let us identify a manometer reading by R_m; in Fig this is the height *OC.* If *y'* is the height of a column of measured fluid that would exert the same pressure at C as does the column of manometer fluid OC, height R_m

  gage pressure $p_c = \Upsilon_M R_m = \Upsilon_F y'$

  and by rearranging

  $y' = (\Upsilon_M / \Upsilon_F) R_m = (p_M / p_F) R_m = (S_M / S_F) R_m$
- Thus the gage pressure at C, in terms of the fluid whose pressure we are mea-suring, as required, is $\Upsilon(S_M / S_F) R_m$ This is also the pressure at *B* because the fluid in *BC* is in balance. The pressure at *A* is
- $0 + \Upsilon(S_M / S_F) R_m + \Upsilon h = P_A$

Or divide all terms by Υ to get the pressure head